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Threshold Behavior of Strongly Localized Nonlinear Modes in Crystals with Fermi Resonance Interaction

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We report the existence of strongly localized modes in thin films and crystals with Fermi resonance interaction between vibration modes of molecules. Several families of bright and dark localized modes are investigated theoretically in two- and three-dimensional systems. Simple theoretical estimations are confirmed by numerical calculations. It is shown that in higher dimensions the solutions exist which do not have their one-dimensional analogs. The strongly localized Fermi resonance modes demonstrate the threshold behavior in any dimension in contrast to the nonlinear lattices with cubic interaction between sites.

Keywords: multilayer molecular structure; Fermi resonance; strongly localized modes

1 INTRODUCTION

For many years there have been attempts to prepare molecular multilayer structures analogous to inorganic superlattices and quantum wells. Recently, such structures have been prepared by molecular beam deposition method and are undergoing intense studies (see, e.g., a review article [1]). These molecular crystalline systems are held together by weak van der Waals forces rather than the valence or ionic forces, giving greater freedom and flexibility in preparing structures which are very promising from the technological as well as the scientific point of view.

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As it was recognized in [2], different types of Fermi resonances can be used as a universal tool for achieving the nonlinear interaction between excitations in these structures. The case of second order nonlinearity, when the molecular vibrations with frequencies ω_a and $\omega_b \simeq 2\omega_a$ interact with each other, seems to be the most important. Prediction of Fermi resonance interface modes [3] in these systems gave rise to the development of the theory of Fermi resonance surface solitons [4]-[7] and nonlinear waves [8] in organic superlattices.

In the above mentioned studies, the width of Fermi resonance solitons was assumed to be much greater than the lattice constant, i.e., the crystalline structure was considered as a continuous medium in the directions along interfaces. However, it is well known that discreteness of nonlinear lattices leads to another important phenomenon–strongly localized modes of excitation [9]-[14]. This phenomenon attracted much attention and was applied to many physical systems (see, e.g., recent review articles [15, 16] and references therein). It is natural to suppose that analogous localized modes can exist in Fermi resonance lattices. Such a supposition was confirmed by numerical calculations for one-dimensional model systems [17]. Similar problem was considered in the case of an array of optical linearly coupled waveguides with second order interaction between the fundamental frequency field and second harmonic field in each of them [18], where several types of localized one-dimensional solutions (symmetric and antisymmetric bright and dark states localized mainly at a single or two sites) were found.

The aim of this article is to study theoretically the strongly localized modes (SLM) in the two- and three-dimensional (2D and 3D) crystalline structures with Fermi resonance interaction between molecules. As we shall see, 2D and 3D systems have SLM similar to those in 1D system, and, in addition, in higher dimensional systems there are new SLM which do not have their 1D counterparts. The most important feature of SLM in the Fermi resonance lattices is that they can only exist if their amplitude (or energy) is greater than some threshold value. The existence of energy thresholds for localized states in nonlinear lattices was supposed and supported by numerical calculations in [19] for lattices with cubic interaction described by the discrete nonlinear Schrödinger equation. We have found that in the Fermi resonance lattices the threshold behavior takes place even in 1D lattices.

2 STRONGLY LOCALIZED MODES IN 2D CRYSTALLINE STRUCTURES

We assume that molecules occupy the sites of a simple square lattice. This lattice may be thought of as a thin film comprised of two monolayers–one made of *a*

molecules and the other of b molecules. Molecular vibrations are described by the complex amplitudes $\mathcal{A}$ and $\mathcal{B}$, correspondingly, and the condition of Fermi resonance $2\omega_a \simeq \omega_b$ between vibration frequencies is fulfilled. The corresponding nonlinear interaction between monolayers is described by the classical Lagrangian

$$L_{FR} = \sum_s \Gamma(\mathcal{A}_s^2 \mathcal{B}_s^* + \mathcal{A}_s^{*2} \mathcal{B}_s), \tag{1}$$

where the sum is taken over all sites s, the star denotes the complex conjugation, and Γ is the nonlinear coupling constant. The excitations can propagate along monolayers according to hopping Lagrangian

$$L_{nn} = \sum_{s,n} [V_a(\mathcal{A}_{s+n}^* \mathcal{A}_s + \mathcal{A}_{s+n} \mathcal{A}_s^*) + V_b(\mathcal{B}_{s+n}^* \mathcal{B}_s + \mathcal{B}_{s+n} \mathcal{B}_s^*)], \tag{2}$$

where $\mathbf{s} + \mathbf{n}$ denotes the vectors to the *nearest neighbors* of the site $\mathbf{s}$, and V_a and V_b are the linear coupling constants of interaction between nearest sites within each monolayer. Harmonic vibrations in each site are described by the Lagrangian

$$L_h = \sum_s \left[\frac{i}{2} (\dot{\mathcal{A}}_s^* \mathcal{A}_s - \mathcal{A}_s^* \dot{\mathcal{A}}_s + \dot{\mathcal{B}}_s^* \mathcal{B}_s - \mathcal{B}_s^* \dot{\mathcal{B}}_s) + \beta \mathcal{B}_s^* \mathcal{B}_s \right], \tag{3}$$

where overdot stands for the time derivative and $\beta = \omega_b - 2\omega_a$ measures the detuning from the exact resonance condition. We look for the periodic solutions

$$\mathcal{A}_s = \exp(-i\Omega t/2) A_s, \quad \mathcal{B}_s = \exp(-i\Omega t) B_s, \tag{4}$$

where the amplitudes A_s and B_s are considered as real variables, and the above Lagrangians permit us to write down readily the equations of motion for any specific configuration.

Strongly localized modes may be expected if the nonlinear terms are much greater than the linear coupling terms responsible for transfer of the excitation energy to neighboring sites. This limit is suitable for analytical treatment and we shall begin with its study. Comparison of (1), (2), and (3) gives us the condition for existence of SLM:

$$\Omega \simeq \Gamma B, \quad |V_a/\Gamma B| \ll 1, \quad |V_b/(\Gamma B - \beta)| \ll 1, \tag{5}$$

where B is the characteristic value of vibration amplitude.

(a) We shall start with the simplest SLM with single strongly excited site which is denoted by index "0" (see Figure 1). The nearest sites denoted by index "1" are excited much weaker, and excitations of the other sites are negligibly small. The equations of motion for sites "0" and "1" read

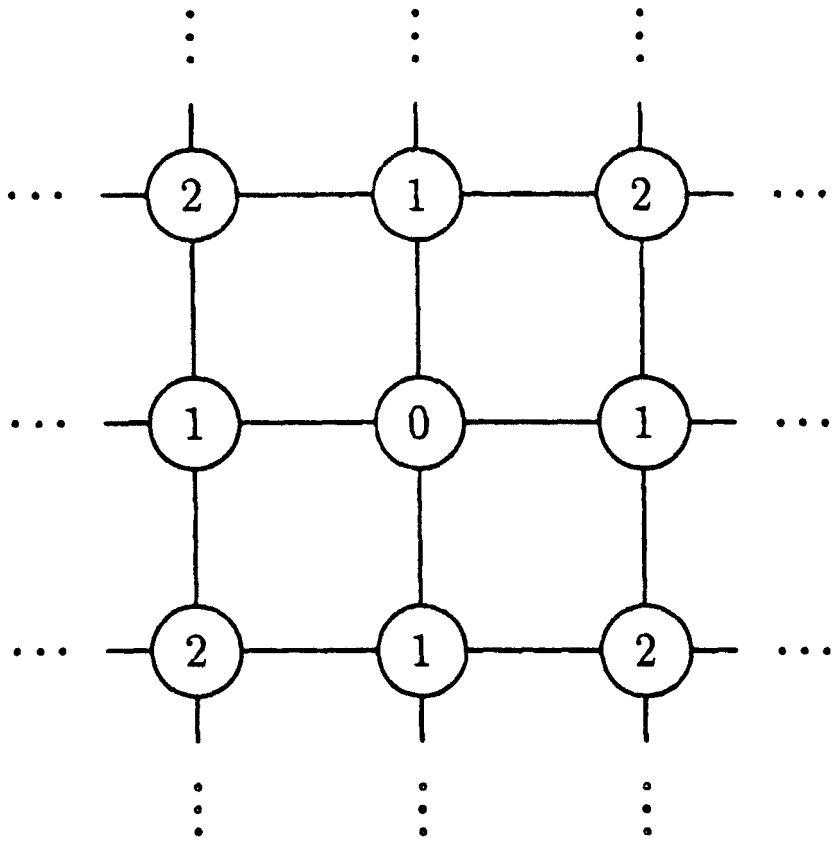

FIGURE 1 Pattern of strongly excited modes in two-dimensional crystal. In case Section 2 (a) the central site "0" is strongly excited, the sites "1" are excited much weaker, and the excitation of sites "2" and others is negligibly small. In case Section 2 (d) the sites "0" and "1" are strongly excited, and the sites "2" and others are excited much weaker. In case of dark SLM (Section 2 (e)) the central cite "0" is excited very weakly, the cites "1" are excited much stronger, and the other sites can be considered as belonging to the homogeneous state

$$\begin{aligned} \Omega A_0/2 &= 4V_a A_1 + 2\Gamma A_0 B_0, \\ (\Omega - \beta)B_0 &= 4V_b B_1 + \Gamma A_0^2, \end{aligned} \tag{6}$$

and

$$\begin{aligned} \Omega A_1/2 &= V_a A_0 + 2\Gamma A_1 B_1, \\ (\Omega - \beta)B_1 &= V_b B_0 + \Gamma A_1^2, \end{aligned} \tag{7}$$

correspondingly. We suppose that

$$A_1 = \rho A_0, \quad B_1 = \mu B_0, \quad |\rho|, |\mu| \ll 1, \tag{8}$$

and, hence, equations (6) give us as the first approximation the relations

$$\Omega \simeq 4\Gamma B_0, \quad A_0^2 \simeq 4B_0^2 - \beta B_0/\Gamma. \tag{9}$$

Then from equations (7) we find the ratios ρ and μ of the amplitudes,

$$\rho \simeq \frac{V_a}{2\Gamma B_0}, \quad \mu \simeq \frac{V_b}{4\Gamma B_0 - \beta}, \tag{10}$$

which are supposed to be small in accordance with (5). The relations (9) and (10) coincide with those for the corresponding SLM in 1D structures [18].

(b) Now let us consider "even" symmetric mode with four strongly localized neighboring sites with a molecules vibrating in phase with each other (see Figure 2). The equations for sites with index "0" have the form

$$\Omega A_0/2 = V_a(2A_0 + 2\rho A_0) + 2\Gamma A_0 B_0,$$
$$(\Omega - \beta)B_0 = V_b(2B_0 + 2\mu\beta) + \Gamma A_0^2,$$

and give in the limit (8) the expressions

$$\Omega \simeq 4\Gamma B_0 + 4V_a, \quad A_0^2 \simeq 4B_0^2 + (4V_a - 2V_b - \beta)B_0/\Gamma. \tag{11}$$

Equations for sites with index "1" have the form

$$\Omega\rho A_0/2 = V_a(A_0 + \rho A_0) + 2\Gamma\rho\mu A_0 B_0,$$
$$(\Omega - \beta) = V_b(B_0 + \mu B_0) + \Gamma\rho^2 A_0^2,$$

where we can neglect the terms quadratic in ρ and μ and obtain the ratios of the amplitudes

$$\rho = \frac{V_a}{2\Gamma B_0 + V_a}, \quad \mu = \frac{V_b}{4\Gamma B_0 + 4V_a - V_b - \beta}. \tag{12}$$

(c) In even antisymmetric mode two a molecules vibrate in opposite phase to vibrations of the other two a molecules of the SLM (see Figure 3), i.e. $A_{-1} = -A_1$, $A_{-2} = -A_2$, etc. It is easy to find that $B_{-1} = B_1$, $B_{-2} = B_2$, etc, and equations of motion for sites "$-n$" coincide with those for sites "n". Thus, from equations for sites "1",

$$\Omega A_1/2 = V_a(-2A_1 + 2A_2) + 2\Gamma A_1 B_1,$$
$$(\Omega - \beta)B_1 = V_b(2B_1 + 2B_2) + \Gamma A_1^2,$$

we find

$$\Omega \simeq 4\Gamma B_1 - 4V_a, \quad A_1^2 \simeq 4B_1^2 - (4V_a - 2V_b - \beta)B_1/\Gamma, \tag{13}$$

and equations for sites "2"

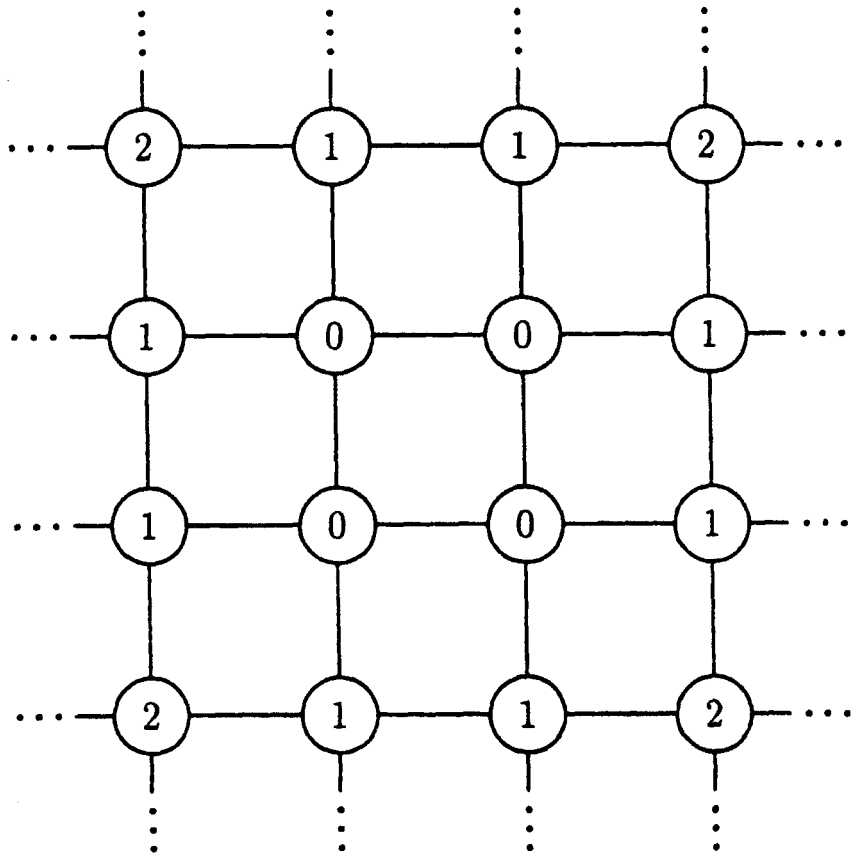

FIGURE 2 Pattern of even symmetric SLM in two-dimensional crystal (Section 2 (b)). The central sites "0" are strongly excited, the sites "1" are excited much weaker, and the excitation of sites "2" and others is negligibly small

$$\Omega\rho A_1/2 = V_a(A_1 - \rho A_1) + 2\Gamma\rho\mu A_1 B_1,$$
$$(\Omega - \beta)\mu B_1 = V_b(B_1 + \mu B_1) + \Gamma\rho^2 A_1^2,$$

yield

$$\rho = \frac{V_a}{2\Gamma B_0 - V_a}, \quad \mu = \frac{V_b}{4\Gamma B_0 - 4V_a - V_b - \beta}, \tag{14}$$

which again are small parameters.

(d) The largest bright SLM we shall consider consists of five sites denoted by indices "0" and "1" in Figure 1. We suppose that

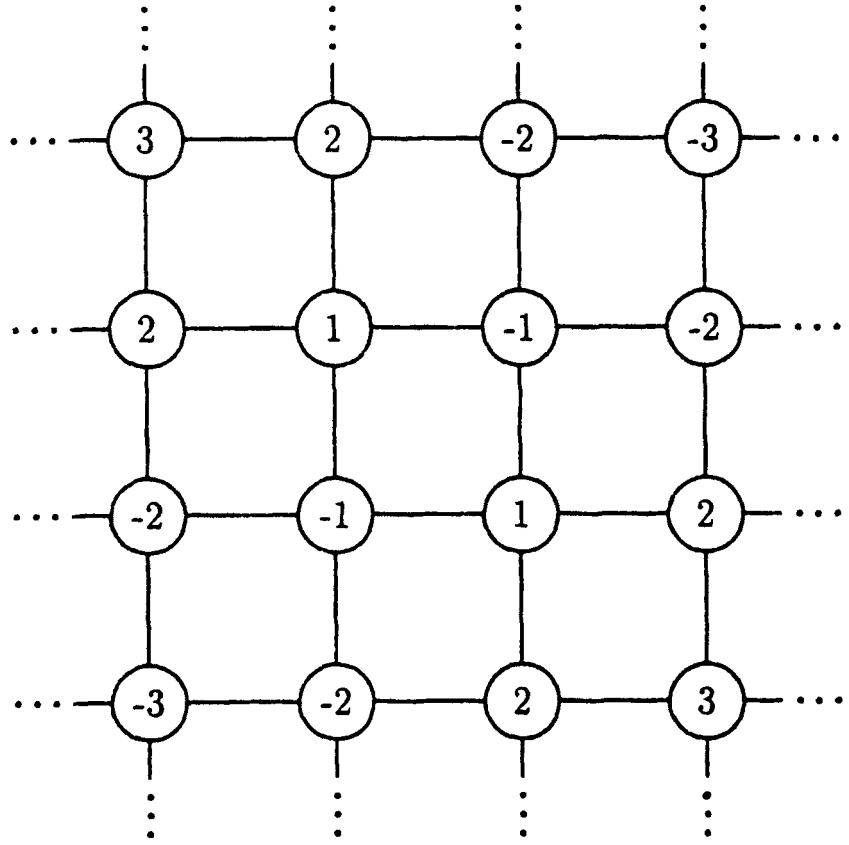

FIGURE 3 Pattern of even antisymmetric SLM in two-dimensional crystal (Section 2 (c)). The sites "1" and "-1" are strongly excited and the corresponding vibration A-amplitudes have opposite signs. The sites "2" and "-2" are excited much weaker, and the excitation of other sites is negligibly small

$$A_1 = (1+\varepsilon)A_0, \quad B_1 = (1+\delta)B_0, \quad A_2 = \rho A_0, \quad B_2 = \mu B_0,$$
$$\varepsilon, \delta, \rho, \mu \ll 1.$$

Equations for the central site "0"

$$\Omega A_0/2 \simeq 4V_a A_0 + 2\Gamma A_0 B_0,$$
$$(\Omega - \beta)B_0 \simeq 4V_b B_0 + \Gamma A_0^2,$$

give the relations

$$\Omega \simeq 4\Gamma B_0 + 8V_a, \quad A_0^2 \simeq 4B_0^2 + (8V_a - 4V_b - \beta)B_0/\Gamma, \tag{15}$$

and equations for sites "1" and "2" yield the values of small parameters

$$\varepsilon \simeq \frac{3V_a}{2\Gamma B_0}, \quad \delta \simeq \frac{3(2V_a + V_b)}{8\Gamma B_0}, \quad \rho \simeq \frac{V_a}{\Gamma B_0} \quad \mu \simeq \frac{V_b}{2\Gamma B_0}. \tag{16}$$

As we see, the central site "0" is excited a little weaker than the neighboring sites "1".

(e) The last 2D structure we consider is the dark SLM. It is easy to find that homogeneous excitation with constant $A_s = A$, $B_s = B$ is described by the equations

$$\Omega A/2 = 4V_a + 2\Gamma AB, \quad (\Omega - \beta)B = 4V_b B + \Gamma A^2,$$

which immediately give

$$\Omega = 4\Gamma B + 8V_a, \quad A^2 = 4B^2 + (8V_a - 4V_b - \beta)B/\Gamma. \tag{17}$$

We look for the solution with one site excited very weakly, i.e. (see Figure 1)

$$A_0 = \rho A, \quad B_0 = \mu B, \quad A_1 = (1 + \varepsilon)A, \quad B_1 = (1 + \delta)B, \quad \rho, \mu, \varepsilon, \delta \ll 1, \tag{18}$$

and the other sites can be considered as belonging to the uniformly excited state. Equations for site "0" have the form

$$\Omega\rho A/2 = 4V_a(1 + \varepsilon)A + 2\Gamma\rho\mu AB,$$
$$(\Omega - \beta)\mu B = 4V_b(1 + \delta)B + \Gamma\rho^2 A^2,$$

and neglecting the second order degrees of small parameters we find

$$\rho = \frac{2V_a}{\Gamma B + 2V_a}, \quad \mu = \frac{4V_b}{4\Gamma B + 8V_a - \beta}. \tag{19}$$

Equations for sites "1"

$$\Omega(1 + \varepsilon)A/2 = V_a(\rho A + 3A) + 2\Gamma(1 + \varepsilon + \delta)AB,$$
$$(\Omega - \beta)(1 + \delta)B = V_b(\mu B + 3B) + \Gamma(1 + 2\varepsilon)A^2,$$

with the use of Eqs. (17) yield the values of the other small parameters

$$\varepsilon \simeq \frac{2V_a + V_b}{8\Gamma B}, \quad \delta \simeq \frac{V_a}{2\Gamma B}. \tag{20}$$

As we see, the nearest neighbors of the central weakly excited site are excited a little stronger than the region with the homogeneous excitation.

Now let us proceed to the 3D structures.

3 STRONGLY LOCALIZED MODES IN 3D CRYSTALS

We assume now that molecules occupy the sites of a simple cubic lattice, and Fermi resonating vibration modes belong to each molecule. Nonlinear Lagrangian (1) describes this intramolecular Fermi resonance interaction, whereas the Lagrangian (2) corresponds to the intermolecular interaction between neighboring sites. All three Lagrangians (1–3) preserve their form in 3D case where it is implied now that **s** and **s + n** are 3D vectors pointed at the sites of a cubic lattice.

We shall consider here three typical structures.

(a) The single strongly excited site "0" has six nearest neighbors and in the first approximation we obtain again the same relations (9, 10) as in 1D and 2D cases. It is explained by the fact that in the first approximation we neglect all interactions of the central site "0" with its neighbors in the equation of motion for this site, so Eqs. (9) do not depend on the dimension of the system. Similarly, in the equation of motion for nearest neighbors "1" we take into account only their interaction with the central site, so the resulting expressions (10) do not depend again on the dimension of the system.

(b) Now let us consider the SLM with strongly excited central site "0" and six nearest neighboring sites "1" (3D counterpart of 2D case (d)). Simple calculation analogous to that in 2D case gives (the meanings of the symbols are the same as in 2D case)

$$\Omega = 4\Gamma B_0 + 12V_a, \quad A_0^2 = 4B_0^2 + (12V_a - 6V_b - \beta)B_0/\Gamma, \tag{21}$$

$$\varepsilon = \frac{5(2V_a + V_b)}{8\Gamma B_0}, \quad \delta = \frac{5V_a}{2\Gamma B_0}, \quad \rho = \frac{V_a}{\Gamma B_0}, \quad \mu = \frac{V_b}{2\Gamma B_0}, \tag{22}$$

which may be compared with Eqs. (15, 16).

(c) At last let us turn to the dark 3D SLM. Now the homogeneous state with constant amplitudes is described by the relations

$$\Omega = 4\Gamma B + 12V_a, \quad A^2 = 4B^2 + (12V_a - 6V_b - \beta)B/\Gamma. \tag{23}$$

Again we look for the solution in the form (18) and simple calculation similar to that in 2D case yields

$$\rho = \frac{3V_a}{\Gamma B + 3V_a}, \quad \mu = \frac{4V_b}{4\Gamma B + 12V_a - \beta}, \quad \varepsilon = \frac{2V_a + V_b}{8\Gamma B}, \quad \delta = \frac{V_a}{2\Gamma B}. \tag{24}$$

It is clear that some other 3D SLM can be considered in the same way. The estimations obtained in this and preceding sections give the order of magnitude of the main parameters of the SLM. Exact results can be obtained numerically by the method presented in the following section.

4 EXACT NUMERICAL METHOD

It is easy to notice that B-amplitudes can be expressed in terms of A-amplitudes by means of equations of motion for A-amplitudes. Substitution of these expressions into equations of motion for B-amplitudes yields the nonlinear system of equations which contains as unknown variables only A-amplitudes and can be easily solved numerically. Formally, this nonlinear system consists of infinite number of equations, but for SLM under consideration it is natural to impose periodic boundary conditions which, practically, do not influence on the SLM solution for large enough lattices. Symmetry consideration can also drastically reduce the number of unknown variables.

We shall illustrate this method in some detail by the simplest example of 2D SLM with single strongly excited site. In this case it is enough to consider 3 by 3 lattice shown in Figure 1 with identified opposite sides, so that equations of motion have the form

$$\begin{aligned}
\Omega A_0/2 &= 4V_a A_1 + 2\Gamma A_0 B_0, \\
\Omega A_1/2 &= V_a(A_0 + A_1 + 2A_2) + 2\Gamma A_1 B_1, \\
\Omega A_2/2 &= V_a(2A_1 + 2A_2) + 2\Gamma A_2 B_2,
\end{aligned} \tag{25}$$

for A-amplitudes, and

$$\begin{aligned}
(\Omega - \beta)B_0 &= 4V_b B_1 + \Gamma A_0^2, \\
(\Omega - \beta)B_1 &= V_b(B_0 + B_1 + 2B_2) + \Gamma A_1^2, \\
(\Omega - \beta)B_2 &= V_b(2B_1 + 2B_2) + \Gamma A_2^2,
\end{aligned} \tag{26}$$

for B-molecules. We express B_0, B_1, B_2 in terms of A_0, A_1, A_2 from Eqs. (25) and substitute these expressions into Eqs. (26). As a result, we obtain the nonlinear system

$$\begin{aligned}
&D - 4GA_1/A_0 + 4H(A_0/A_1 + 1 + 2A_2/A_1) = A_0^2, \\
&D - G(A_0/A_1 + 1 + 2A_2/A_1) \\
&+ H(4A_1/A_0 + A_0/A_1 + 5 + 2A_2/A_1 + 4A_1/A_2) = A_1^2, \\
&D - 2G(A_1/A_2 + 1) + 2H(A_0/A_1 + 3 + 2A_2/A_1 + 2A_1/A_2) = A_2^2,
\end{aligned} \tag{27}$$

where the parameters

$$D = \frac{\Omega(\Omega - \beta - 4V_b)}{4\Gamma^2}, \quad G = \frac{V_a(\Omega - \beta)}{2\Gamma^2}, \quad H = \frac{V_a V_b}{2\Gamma^2} \tag{28}$$

have been introduced. The calculations of the preceding section correspond to the supposition that the following inequalities are satisfied

$$D \gg G \gg H \simeq 1. \tag{29}$$

Let us introduce the ratios of amplitudes

$$\rho_1 = \frac{A_1}{A_0}, \quad \rho_2 = \frac{A_2}{A_1}; \tag{30}$$

then Eqs. (27) lead to the following system for ρ_1, ρ_2:

$$\begin{aligned}
&[D - 4G\rho_1 + 4H(1/\rho_1 + 1 + 2\rho_2)]\rho_1^2 = D - G(1/\rho_1 + 1 + 2\rho_2) \\
&+ H[(1/\rho_1 + 1 + 2\rho_2) + 4(1/\rho_2 + 1 + \rho_1)], \\
&\{D - G(1/\rho_1 + 1 + 2\rho_2) + H[(1/\rho_1 + 1 + 2\rho_2) + 4(1/\rho_2 + 1 \\
&+ \rho_1)]\}\rho_2^2 = D - 2G(1/\rho_2 + 1) + 2H[1/\rho_1 + 1 + 2\rho_2 + 2(1/\rho_2 + 1)].
\end{aligned} \tag{31}$$

It is easy to find approximate solution to these equations corresponding to estimation of case (a) of section 2. Neglecting terms proportional to small parameter H, we reduce Eqs. (31) to the system

$$\begin{aligned}
(D - 4G\rho_1)\rho_1^2 &= D - G(1/\rho_1 + 1 + 2\rho_2), \\
[D - G(1/\rho_1 + 1 + 2\rho_2)]\rho_2^2 &= D - 2G(1/\rho_2 + 1).
\end{aligned} \tag{32}$$

We look for the solution with

$$\rho_1 \ll 1, \quad \rho_2 \ll 1,$$

so that Eqs. (32) give immediately

$$\rho_1 \simeq \frac{G}{D}, \quad \rho_2 \simeq \frac{2G}{D},$$

which actually coincides with Eqs. (10).

For analogous estimation of other SLM the 3 by 3 lattice with periodic boundary conditions is too small and we have to involve into consideration either 5 by 5 or 7 by 7 matrices with increasing number of unknown variables in the corresponding nonlinear systems of equations. These systems were solved numerically and the results are shown in figures 4–6 for 1D, 2D, and 3D cases, correspondingly. In these figures the A-amplitudes are plotted as functions of the distance from the central site "0" for the cases of single excited site, i.e. case (a) of sections 2 and 3, (Figure 4); three (1D), five (2D), or seven (3D) excited states, i.e. cases (d) of section 2 and (b) of section 3, (Figure 5); and dark SLM, i.e. cases (e), section 2, and (c), section 3, (Figure 6) with their 1D analogs. To show better the fine structure of these modes, the ordinate axes correspond to the A variable in the power (1/4). The plots were calculated at the values of the parameters: $\Omega/\omega_b = 1.7$, $V_a/\omega_b = V_b/\omega_b = 0.01$, $(\omega_b - 2\omega_a)/\omega_b = 0.2$.

The value of frequency Ω must be large enough to satisfy the conditions (29). In fact, numerical calculations have shown that Ω should be greater than some threshold value to lead to the SLM under consideration, see Figure 7. This means, because of the relation $\Omega \simeq 4\Gamma B \simeq 2\Gamma A$, that the localized modes exist

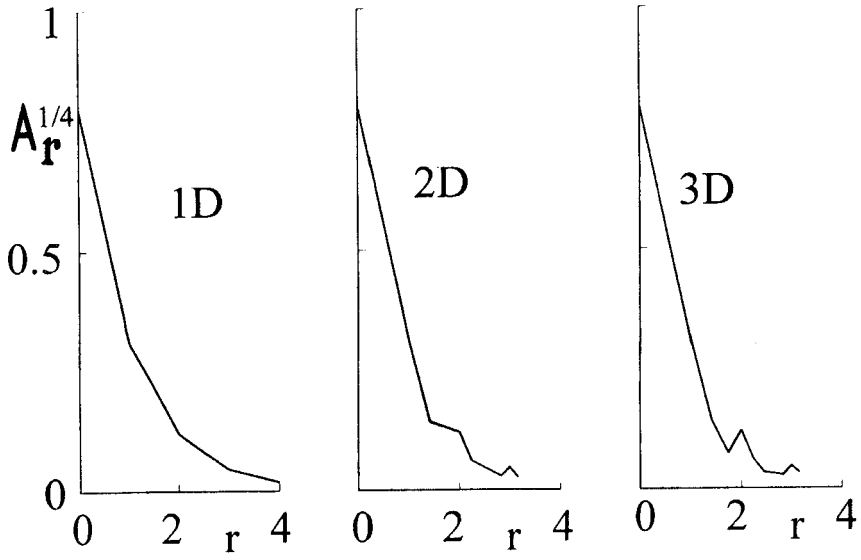

FIGURE 4 The dependence of vibration amplitudes of SLM with single excited site (cases (a) in sections 2 and 3) on the distance from the central site. The chosen values of the physical parameters of the lattice are equal to $V_a/\omega_b = V_b/\omega_b = 0.01$, $(\omega_b - 2\omega_a)/\omega_b = 0.2$; the frequency $\Omega/\omega_b = 1.7$ is chosen above the threshold value for all three dimensions

only at large enough excitation amplitudes what justifies the use of classical approximation. The threshold values of Ω depend on the dimension of the lattice and are equal to in 1D lattices to $\Omega/\omega_b \simeq 1.02$, in 2D lattices to $\Omega/\omega_b \simeq 1.14$, and in 3D lattices to $\Omega/\omega_b \simeq 1.54$ for above choice of the parameters. It is interesting that in Fermi resonance lattices such threshold behavior takes place even in 1D case in contrast to nonlinear lattices with cubic interaction between molecules where thresholds exist only for higher (2D and 3D) dimensions [19].

The fine oscillating structure at dimensions 2 and 3 is caused by geometrical reasons, namely, by the fact that the site located closer to the central one can have weaker excited surrounding than some other sites situated at larger distances from the central site. For example, in cubic lattice with SLM shown in Figure 5 the site at distance $\sqrt{3}$ interacts with weakly excited sites "2" whereas the site at distance 2 interacts with strongly excited site "1", and this circumstance is responsible for the sharp minimum in this plot at $\sqrt{3} \simeq 1.7$.

In general, numerical calculations strongly support the estimations of preceding sections and justify simple method by which they were obtained.

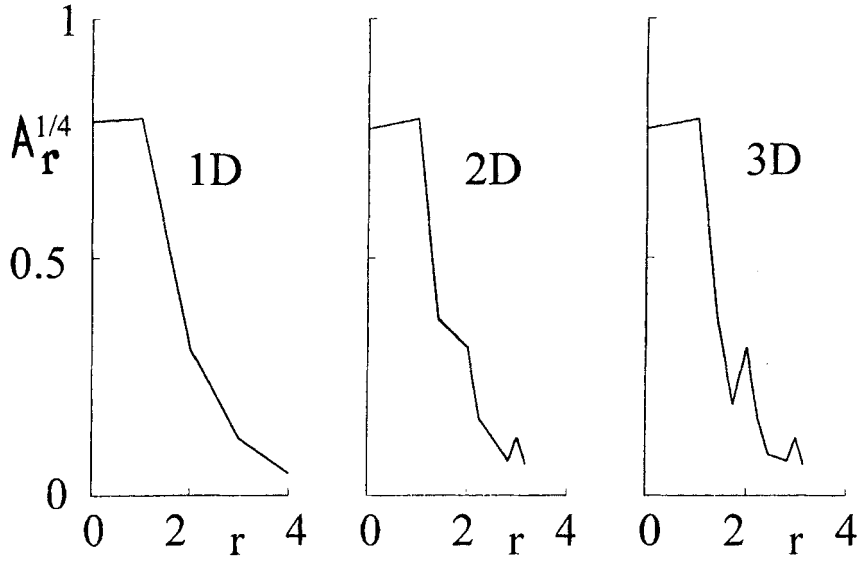

FIGURE 5 The same as in figure 4, but for "large" bright SLM (cases (d) in section 2 and (b) in section 3)

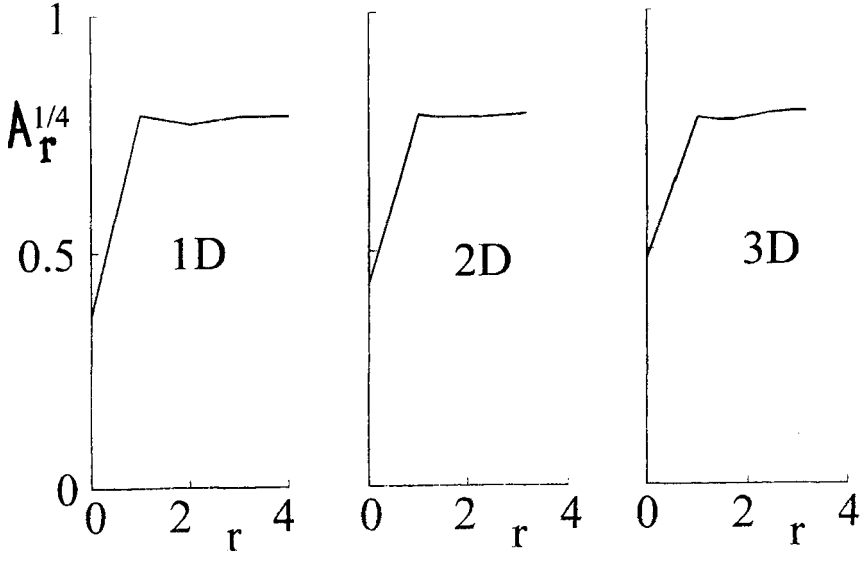

FIGURE 6 The same as in figure 4, but for dark SLM (cases (e) in section 2 and (c) in section 3)

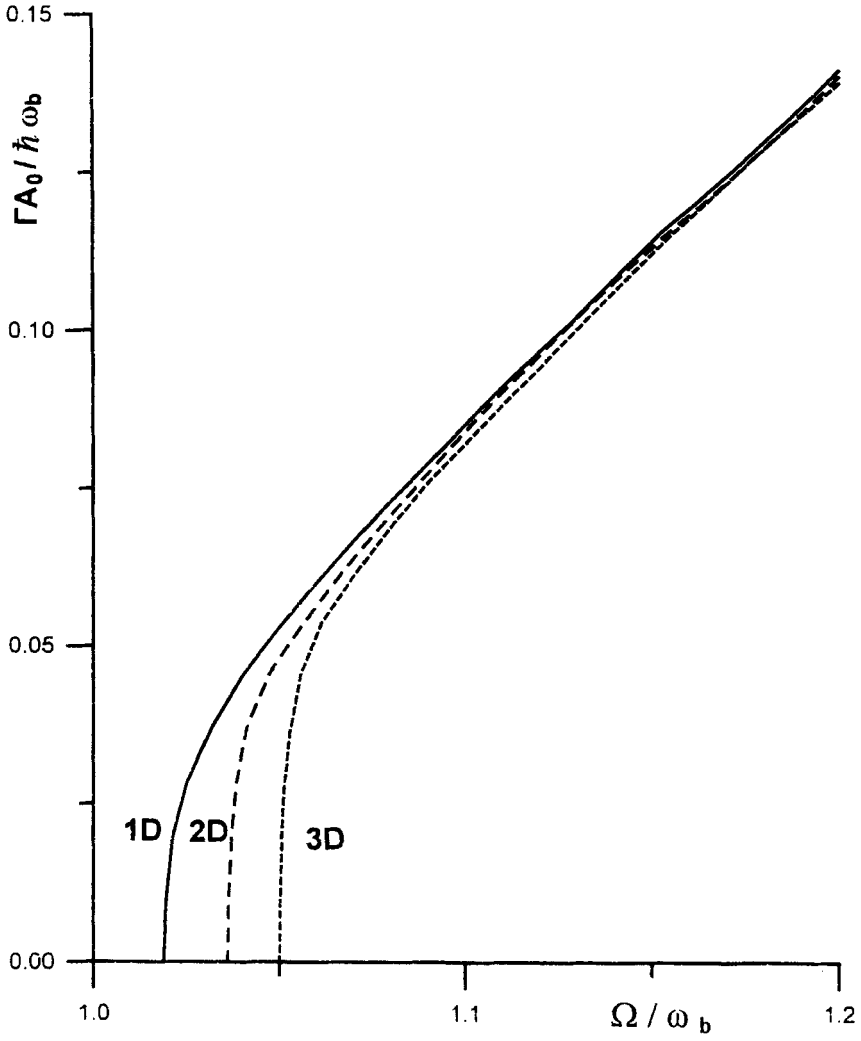

FIGURE 7 The dependence of the amplitude of the central site for the SLM modes shown in Figure 4 on the frequency Ω

5 CONCLUSION

We have shown that in Fermi resonance crystalline structures several families of bright and dark strongly localized modes can exist. Two- and three-dimensional structures have been investigated which can play an important role in the proc-

esses of energy transformation in thin films and crystals with Fermi resonance interaction between vibration modes. The threshold nonlinear dynamics of SLM is found. One can suppose that similar localized modes exist in Fermi resonance systems with electronic or vibronic excitations and in optical systems with second-order nonlinearity.

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